

TABLE II. Observed and calculated values of  $\partial T_c/\partial P$ .

Element	$T_c$ °K	Atomic volume cm <sup>3</sup>	$(C_s - C_n)_{T=T_c}$ mJ deg <sup>-1</sup> mole <sup>-1</sup>	$\left(\frac{\partial H_c}{\partial T}\right)_{P=0, T=T_c}$ Oe deg <sup>-1</sup>	$H_0$ Oe	$\left(\frac{\partial H_c}{\partial P}\right)_{T=T_c}$ 10 <sup>-3</sup> Oe bar <sup>-1</sup>	$\left(\frac{\partial T_c}{\partial P}\right)_{H=0}^{\text{calc}}$ 10 <sup>-6</sup> deg bar <sup>-1</sup>	$\left(\frac{\partial T_c}{\partial P}\right)_{H=0}^{\text{obs}}$ 10 <sup>-6</sup> deg bar <sup>-1</sup>
V	5.03 <sup>a</sup>	8.34	69.4 <sup>a</sup>	-455	1310 <sup>a</sup>	4.1±0.3 <sup>b</sup> 2.0±0.2 <sup>c</sup>	9.0 4.4	11±3
Nb	9.17 <sup>d</sup>	10.80	140 <sup>d</sup>	-421±4	1944 <sup>d</sup>	-1.2±0.3 <sup>e</sup>	-2.85	0±3
Ta	4.39 <sup>e</sup>	10.83	42.2 <sup>e</sup>	-334±2	825 <sup>f</sup>	-0.8±0.3 <sup>e</sup>	-2.4	-2.6±1.0 <sup>f</sup>

<sup>a</sup> Reference 5.<sup>b</sup> Reference 10.<sup>c</sup> Reference 8.<sup>d</sup> Reference 7.<sup>e</sup> Reference 6.<sup>f</sup> Reference 9.

in Table II. In order to calculate values of  $\partial T_c/\partial P$  using the Maxwell thermodynamic relationship,<sup>4</sup>

$$\left(\frac{\partial T_c}{\partial P}\right)_{H=0} = -\left(\frac{\partial H_c}{\partial P}\right)_{T=T_c} \left(\frac{\partial H_c}{\partial T}\right)_{P=0}^{-1}, \quad (3)$$

we express the measured values<sup>5-7</sup> of  $C_s - C_n$  in terms of  $(\partial H_c/\partial T)_{T=T_c}$  using the Rutgers relationship,<sup>4</sup>

$$(C_s - C_n)_{T=T_c} = \frac{VT_c}{4\pi} \left(\frac{\partial H_c}{\partial T}\right)_{P=0}^2. \quad (4)$$

The values of  $\partial H_c/\partial T$ , given in Table II, derived in this manner are in good agreement with values obtained from directly measured critical-field curves for vanadium<sup>5</sup> and tantalum,<sup>6</sup> but not for niobium.<sup>7</sup>

Using the thermodynamic relationship (4) we have calculated values of  $(\partial T_c/\partial P)_{H=0}$ , and these are compared in Table II with our observed values. Table II also includes the results for tantalum;  $(\partial T_c/\partial P)_{H=0}$  was determined for this element by Hinrichs and Swenson.<sup>9</sup> The sign of  $(\partial T_c/\partial P)_{H=0}$  obtained for vanadium agrees with that predicted from the thermal-expansion data. The observed magnitude is in better agreement with the value calculated from the thermal-expansion data of Müller and Rohrer,<sup>10</sup> rather than the value determined from the data of White.<sup>8</sup> The calculated value of  $(\partial T_c/\partial P)_{H=0}$  for niobium is about the limit of our experimental sensitivity and is, therefore, not inconsistent with the zero pressure dependence observed. The experimental results of Hinrichs and Swenson<sup>9</sup> are also in good agreement with the calculated value.

The effect of applying pressure to a superconductor, until recently, had always been associated with an observed decrease in the superconducting transition temperature.<sup>11</sup> However, a number of superconductors

(Zr,<sup>12</sup> La,<sup>13</sup> U,<sup>14</sup> and V<sup>15</sup>) have now been found to exhibit a positive  $\partial T_c/\partial P$ . We may attempt to understand this difference in sign of the pressure dependence of the superconducting transition temperature by considering the volume derivative of the BCS<sup>16</sup> relationship,

$$T_c = 0.85\Theta_D \exp(-1/A), \quad (5)$$

with  $A = N(0)V$ , where  $N(0)$  is the density of electron states at the Fermi surface and  $V$  is the attractive electron-electron interaction parameter. Differentiation of (5) with respect to volume gives

$$\frac{\partial \ln T_c}{\partial \ln v} = \varphi \ln\left(\frac{0.85\Theta_D}{T_c}\right) - \gamma_G, \quad (6)$$

where  $\varphi = \partial \ln A/\partial \ln v$  and  $\gamma_G$ , the Grüneisen constant, represents the volume dependence of the phonon spectrum. Rewriting  $\partial \ln T_c/\partial \ln v$  in terms of  $\partial T_c/\partial P$  we have

$$\frac{\partial T_c}{\partial P} = -|K|T_c \left\{ \varphi \ln\left(\frac{0.85\Theta_D}{T_c}\right) - \gamma_G \right\}, \quad (7)$$

where  $K$  is the compressibility.

The pressure dependence of the phonon spectrum is such as to increase  $T_c$  and will be roughly the same for all elements since  $\gamma_G$  has, in general, values between 1 and 3. Since  $\ln(0.85\Theta_D/T_c)$  lies in the range 2.5 to 6.5 for most superconductors the sign and magnitude of  $\partial T_c/\partial P$  is determined by  $\varphi$ . Rohrer<sup>17</sup> has pointed out that for nontransition metal superconductors  $\varphi$  is roughly constant and equal to  $2.5 \pm 0.5$ . However, when we consider the behavior of the transition metal superconductors there is considerable variation both in the magnitude and the sign of  $\varphi$ .<sup>18,19</sup> Olsen and his co-

<sup>12</sup> N. B. Brandt and N. I. Ginzburg, Zh. Eksperim. i Teor. Fiz. 46, 1212 (1964) [English transl.: Soviet Phys.—JETP 19, 823 (1964)].

<sup>13</sup> W. E. Gardner and T. F. Smith, Phys. Rev. 138, A484 (1965).

<sup>14</sup> T. F. Smith and W. E. Gardner, Phys. Rev. 140, A1620 (1965).

<sup>15</sup> Present work.

<sup>16</sup> J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

<sup>17</sup> H. Rohrer, Helv. Phys. Acta 33, 675 (1960).

<sup>18</sup> J. L. Olsen, E. Bucher, M. Levy, J. Müller, E. Corenzwit, and T. Geballe, Rev. Mod. Phys. 36, 168 (1964).

<sup>19</sup> E. Bucher, J. Müller, J. L. Olsen, and G. Palmy, Phys. Letters 15, 303 (1965).

<sup>9</sup> C. H. Hinrichs and C. A. Swenson, Phys. Rev. 123, 1106 (1961).

<sup>10</sup> J. Müller and H. Rohrer, Helv. Phys. Acta 31, 289 (1958).

<sup>11</sup> An exception to this generalization is thallium which shows a slight increase in  $T_c$  for applied pressures up to 2 kbar [Ref. 1; J. Hattton, Phys. Rev. 103, 1167 (1956); and I. D. Jennings and C. A. Swenson, Phys. Rev. 112, 31 (1958)]. Further application of pressure then causes  $T_c$  to decrease. Jennings and Swenson have explained this behavior as a consequence of the highly anisotropic nature of the physical properties of thallium.